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Self-∗ Distributed Query Region Covering in Sensor Networks

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In this paper, we design two novel *self-∗* solutions to the minimal connected sensor cover problem. The concept of *self-∗* is used to include fault-tolerant properties like *self-configuring*, *self-reconfiguring*, *self-healing*, etc. We present two *self-stabilizing, fully distributed, strictly localized, and scalable* solutions, and show that these solutions are both self-configuring and self-healing. In a self-stabilizing system, every computation, starting from an arbitrary state (even a faulty state), eventually reaches a state which satisfies the specification. Nodes achieve the global objective by using only local computations. Local algorithm based sensor networks are more robust, and scale better. The proposed solutions are space optimal in terms of the number of states used per node. Another feature of the proposed algorithms is that the faults are contained only within the neighborhood of the faulty nodes. This paper also includes a comparison of the performance of the two proposed solutions in terms of the stabilization time, cover size metrics, and ability to cope with transient and permanent faults.

I. Introduction

AFTER spending the first era of computing with mainframes, we are now in the era of personal computing. The next wave, the third era of computing was visioned by Late Mark Weiser. In 1988 at the Computer Science Lab at Xerox PARC, he articulated the next age of computing, called *ubiquitous computing*.[1] In ubiquitous world, we expect to see thousands of invisible computing devices used per person, maybe, even in a household. We can now build ad-hoc networks composed of a large number of low-cost, low-power, and small sensor nodes. These ad-hoc *wireless sensor networks*[2] have applications everywhere—military, business, commercial, health, and home.

Sensor networks[3] are expected to be very large. In many applications, they will be densely deployed. These networks are energy constrained. Not only the sensors have limited battery power, it is extremely difficult if not impossible to replace the battery. They may be deployed in inaccessible terrains or disaster areas. So, it is very important to design energy efficient sensor networks to enable untethered and unattended operation for an extended

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period of time. The topology may change very frequently due to various reasons, like position, reliability, available energy, malfunctioning, etc. Thus, designing reliable wireless sensor networks is challenging.

Deploying pre-configured network of a huge number of sensors is impractical. Expecting to be able to manually maintain that size of a network is absurd. Considering all these constraints, the sensor network must be *self-configuring* and *self-maintaining* or *self-healing*.[4] A system is considered to be *self-configuring* if starting from an arbitrary state and an arbitrary input, the system will eventually satisfy the specification or start behaving properly. A *self-healing* system automatically recovers from different perturbations and dynamic changes. A self-healing system can also be characterized as a *self-maintaining* system.

Software systems are being used for almost all business-critical applications. Thus, the availability of these systems is extremely important. The system must be able to adjust to different inputs, adapt to all possible changes of the environment, and handle different faults. The different concepts or terms encapsulated in *self-*$\ast$ have been introduced to characterize different ways of detecting, adjusting, and recovering from the above situations. Unfortunately, these terms have not been formalized in a coherent manner. According to,[5] a self-$\ast$ system should be self-configuring, self-organizing, self-tuning, self-healing, and self-managing. Human organization (specifically, corporate structure) was used as an analogy to explain the concept of self-$\ast$ in.[6] The insights gained from the human organizations combined with concepts from AI and storage systems were used to design self-$\ast$ storage systems.[5]

In this paper, we will present two *self-stabilizing* solutions to an important energy saving problem in sensor networks. Then we will show that these solutions can also be considered as self-$\ast$ solutions. In a self-stabilizing system, every computation, upon starting from an arbitrary state, eventually reaches a state from where the computation satisfies the specification. The paradigm of self-stabilization, introduced by Dijkstra in 1974,[7] is considered to be the most unified strategy to design fault-tolerant systems. Although it is intended to handle transient faults (e.g., memory errors, message omissions/duplications, program counter corruptions), it has been established that almost all types of faults can be dealt with in a stabilizing manner. Readers can refer to[8] for an overview of self-stabilization.

Motivation and Related Work. In sensor networks, *queries* are sent from some devices (could be a satellite, PDA, laptop, or any computer) to sense some data/events over some time period and a geographical region, called a *query region*. A query could be like "Every I ms for the next Y seconds, tell me how many vehicles of type T are moving in direction D in region R". The query region is usually a subset of the total region covered by all the sensors in the network. Considering the limited energy available, one of the most important goals in any protocol on sensor networks is to save energy. Since the sensors are usually densely deployed, there are usually a lot more sensors than required to process a given query. One possible way to minimize usage of energy is not to keep all sensor nodes fully active all the time. Some of them can be put in passive mode some times while others are active in sensing the data in the environment. However, for the sensor networks to be effective, the active nodes must be able to cover the whole query region and maintain the network connectivity at all times.

In,[9] a new optimization problem in sensor networks, called the *connected sensor cover* was introduced to model the query response system. The problem can be informally defined as follows: given a query over a sensor network, select a minimum or nearly minimum set of sensors, called connected sensor cover, such that the selected sensors cover the query region, and form a connected network among themselves. In its general form, this problem is known to be NP-hard.[9,10]

In this work, we address the minimal connected sensor cover problem (introduced in[9]) that constructs a minimal cover with respect to the inclusion operation. Coverage and connectivity issues were addressed in literature with different assumptions: centralized, distributed using some coordinators, and probabilistic. An overview of these approaches is presented below. Two self-organizing solutions were presented in.[9] None of the solutions is localized. The first solution is centralized. In the second solution, a particular sensor node behaves as the coordinator or leader. This special node collects all the global information related to the possible new sensor nodes to be added, then decides which ones to choose.

The issues of coverage and connectivity, and the relations between them were analyzed in a unified framework in.[11] The CCP protocol[11] can be used to provide different degrees of coverage. It was shown in[11,12] that if the communication range is at least twice of the sensing range, the complete coverage implies connectivity. When the above condition does not hold, CCP was integrated with SPAN[13] to provide both coverage and connectivity. However, in SPAN, the nodes need to maintain information about two-hop neighborhood. SPAN is a connectivity maintenance protocol where a node volunteers to be a coordinator when it finds that two of its neighbors cannot communicate with each other directly or indirectly. After a node decides to be a coordinator, it announces that with a random delay to reduce the number of redundant coordinators. A similar approach was discussed in ASCENT.[14] ASCENT nodes use the number of active neighbors and message losses to decide if they should be active or passive. However, this protocol does not guarantee complete coverage of the query region.

Probabilistic studies related to coverage and connectivity in unreliable sensor networks were done in.[15] A sensor grid network of unit area was considered. This work includes a necessary and sufficient condition for the network to remain covered and connected in terms of the probability of a node to be active (i.e., not failed) and transmission radius of the nodes. Some optimal conditions for coverage were established in.[12] An algorithm for coverage was proposed based on those optimal conditions. However, that result is valid only when complete coverage implies connectivity (as discussed above). A coverage protocol using a random delay to announce decision to turn off was proposed in.[16] The issue of connectivity was not addressed in.[16] The GAF protocol[17] uses GPS to reduce the redundant nodes to maintain routing paths in ad-hoc networks. A randomized probing-based density control algorithm was used to maintain coverage under node failures in PEAS protocol.[18] The probing range can be changed to provide different degrees of coverage.

Very recent solutions to the connected cover problem address the fault-tolerance issues by reinforcing the coverage and connectivity degree. Hence in,[19] the authors address the problem of k-coverage. That is, they compute a coverage such that any sensor is covered by k other sensors. This work is further extended in[20] to the k-coverage and k-connectivity problem. The proposed solution involves the computation of a Voronoi diagram for independent sensor nodes. The implementation of local Voronoi diagrams or handling of the transient faults is not addressed.

Contributions. None of the above mentioned solutions explicitly addresses the transient faults (memory and program counter corruptions). The main contribution of this research is to design self-stabilizing solutions to the connected sensor cover problem. To the best of our knowledge, these are the first localized, distributed, and self-stabilizing solutions to the query connected cover problem in sensor networks. Localized solutions in large networks are desirable due to their high reliability and scalability. Self-stabilization[8] is the most adapted theoretical toolkit for the design of algorithms that cope with a broad range of faults in dynamic and large scale networks. Our solutions can handle different types of faults including node and link (wireless communication) failures, change of power level, topology changes due to faults or new joinings, and memory and program counter corruptions. In the next section, we define the model and specify the connected sensor cover problem. In Sections IV and V, we present self-stabilizing solutions to the problem. The proof of the self-$*$ aspects of our solutions is presented in Section VI. Discussion about the complexity of the algorithms and simulation results are included in Section VII. The experiments were conducted with respect to the following metrics: self-stabilization time, cover size, and fault-tolerance. Finally, in Section VIII, we present some concluding remarks and give some ideas to extend this research.

II. Preliminaries and Model

Sensor Network. In this research, we consider a uniform *sensor networks*[9,11] consisting of a large number of sensors (also referred as *sensor nodes* and *nodes* in this paper) randomly distributed in a geographical region. We model the sensor network as a *directed* communication graph $G(V, E)$, where each node in V represents a sensor, and each edge $(i, j) \in E$, called *communication edge*, indicates that j is a *neighbor* of i.

For a sensor i, there is a region, called *sensing region*, which signifies the area in which the sensor i can sense a given physical phenomenon maintaining a desired confidence level. The *sensing range* of a sensor i indicates the maximum distance between i and any point p in the sensing region of i. A point p is *covered* (or monitored) by a sensor node i if the Euclidean distance between p and i is less than the sensing range of i.

The *communication region* of a sensor i (also called the *transmission region*) defines the area in which i can communicate directly (i.e., in single hop) with other sensor nodes. The maximum distance between node i and any other node j, where j is in the communication region of i, is called the *communication range* of i. A directed path (sequence) of sensors $i = i_1, i_2, \ldots, i_m = j$, where i_x is a neighbor of i_{x+1} for $1 \leq x \leq m - 1$, is called a *communication path* from i to j. The length of the shortest (communication) path (which is the number of sensors on the shortest path) from i to j is called the *communication distance* from i to j.

Program. We consider the local shared memory model of communication as used by Dijkstra.[7] The program of every processor consists of a set of *shared variables* (henceforth, referred to as variables) and a finite set of actions. It can only write to its own variables, and read its own variables and variables owned by the neighboring nodes.

Each action is of the following form: $\langle label \rangle :: \langle guard \rangle \longrightarrow \langle statement \rangle$. The guard of an action in the program of p is a boolean expression involving the variables of p and its neighbors. The statement of an action of p updates one or more variables of p. An action can be executed only if its guard evaluates to true. We assume that the actions are atomically executed, meaning, the evaluation of a guard and the execution of the corresponding statement of an action, if executed, are done in one atomic step.

The *state* of a node is defined by the values of its variables. The *state* of a system is the product of the states of all nodes. We will refer to the state of a node and system as a (*local*) *state* and (*global*) *configuration*, respectively. Let a distributed protocol $\mathcal{P}$ be a collection of binary transition relations denoted by $\mapsto$, on $\mathcal{C}$, the set of all possible configurations of the system. A *computation* of a protocol $\mathcal{P}$ is a *maximal* sequence of configurations $e = \gamma_0, \gamma_1, \ldots, \gamma_i, \gamma_{i+1}, \ldots,$ such that for $i \geq 0$, $\gamma_i \mapsto \gamma_{i+1}$ (a single *computation step*) if γ_{i+1} exists, or γ_i is a terminal configuration. The *Maximality* means that the sequence is either infinite, or it is finite and no action of $\mathcal{P}$ is enabled in the final configuration. All computations considered in this paper are assumed to be maximal. The set of all possible computations of $\mathcal{P}$ in system S is denoted as $\mathcal{E}$. A node p is said to be *enabled* in γ $(\gamma \in \mathcal{C})$ if there exists an action A such that the guard of A is true in γ. We consider that any node p executed a *disable action* in the computation step $\gamma_i \mapsto \gamma_{i+1}$ if p was enabled in γ_i and not enabled in γ_{i+1}, but did not execute any action between these two configurations. (The disable action represents the following situation: At least one neighbor of p changed its state between γ_i and γ_{i+1}, and this change effectively made the guard of all actions of p false.) Similarly, an action A is said to be enabled (in γ) at p if the guard of A is true at p (in γ).

We assume a *weakly fair and distributed daemon*. The *weak fairness* means that if a node p is continuously enabled, then p will be eventually chosen by the daemon to execute an action. The *distributed* daemon implies that during a computation step, if one or more nodes are enabled, then the daemon chooses at least one (possibly more) of these enabled nodes to execute an action.

Fault Model. This research deals with the following types of faults: (i) The state or configuration of the system may be arbitrarily corrupted. That is, the nodes memory and counter program can be corrupted. However, the program (or code) of the algorithm cannot be corrupted. (ii) Nodes may crash. That is, the faults can fail-stop nodes. (iii) Nodes may recover or join the network. The topology (both actual and logical topologies) may change due to faults. Faults may occur in any finite number, in any order, at any frequency, and at any time.

Self-stabilization.[8] Let $\mathcal{L}_{\mathcal{A}}$ be a non-empty *legitimacy predicate* of an algorithm $\mathcal{A}$ with respect to a specification predicate *Spec* such that every configuration satisfying $\mathcal{L}_{\mathcal{A}}$ satisfies *Spec*. Algorithm $\mathcal{A}$ is *self-stabilizing* with respect to *Spec* iff the following two conditions hold:

1. Every computation of $\mathcal{A}$ starting from a configuration satisfying $\mathcal{L}_{\mathcal{A}}$ preserves $\mathcal{L}_{\mathcal{A}}$ (*closure*).
2. Every computation starting from an arbitrary configuration contains a configuration that satisfies $\mathcal{L}_{\mathcal{A}}$ (*convergence*).

III. Problem and Description

Our research is focused on designing a reliable self-stabilizing, self-organizing, and self-healing query-response system. A query in sensor networks asks for some data/measurements/events sensed/observed over some period of time at some frequency over a geographical region. Upon receiving a query, the sensors will sense or measure the data and collaborate among themselves to disseminate or fuse the collective data to the sink of the query. Although a query can be initiated in the whole geographical region, typically, a query refers to a subset of the region, called the *query region* denoted by R_Q in this paper.

The sensors only inside the query region should be involved in generating the response to the query. Considering the redundancy and our goal of designing an efficient query-response system, all sensors inside the query region should not be actively participating in the protocol to answer the query. Our approach is for the sensors inside R_Q to self-organize to form a logical network sufficient enough *to cover* the query region. However, in order for the sensors in the logical network (i.e., the region covered by the selected sensors) to be able to collaborate to detect the events, and compute and deliver the response, they must be able to communicate with each other directly or indirectly. That is, the logical graph not only needs to satisfy the coverage criterion, it must also be a strongly connected communication graph.

Definition 3.1 (Connected Sensor Cover). *Consider a sensor network G consisting of n sensors $I_1, I_2, \ldots, I_n$. Let S_i be the sensing region associated with sensor I_i. Given a query Q over a region R_Q in the sensor network, a set of sensors $\mathcal{SC}_Q = I_{i_1}, I_{i_2}, \ldots, I_{i_m}$ is called a* connected sensor cover *for Q if the following two conditions are satisfied:* **(a) Coverage:** $R_Q \subseteq (S_{i_1} \cup S_{i_2} \cup, \ldots, S_{i_m})$. **(b) Connectivity:** *The subgraph induced by $\mathcal{SC}_Q$ is strongly connected in the sense that any two sensors in this set can communicate with each other directly or indirectly.*

Definition 3.2 (Minimal Connected Sensor Coverage Problem). *Given a sensor network and a query over the network, the connected sensor coverage problem is to find the minimal connected sensor cover (we will call it $\mathcal{MCSC}_Q$). A cover is considered minimal if it does not include another connected cover.*

Additionally, we require the algorithm (solving the above problem) to be self-organizing, self-healing, and self-stabilizing.[4,8] That is, regardless of the initial state (wrong initialization of the local variables, memory or program counter corruptions) nodes self-configure/self-organize using only local information in order to make the system self-stabilize to a *legitimate state*. The legitimate state is defined with respect to a minimal connected cover formed out of the nodes that can communicate with each other either directly or indirectly. The nodes in this set are the only nodes that remain active. Moreover, under various perturbations, such as node joins, failures (due to crash or energy loss), state corruptions, or weakening of power, the minimal connected cover should be able to self-heal without any external intervention and the impact should be confined within a tightly bounded region around the perturbed area.

In this paper, we will present two space optimal self-stabilizing, self-configuring and self-healing solutions to the connected sensor cover problem. Computing a minimum sensor cover in its general form is NP-hard.[9,10] So, the proposed solutions make an attempt to approach an optimal solution by checking and removing redundant sensor nodes from the final cover set. However, the solutions although suboptimal in terms of the number of sensors, satisfy Definition 3.2.

Note that removing redundancy while constructing the distributed connected covers is a difficult task, and its degree of accuracy depends on the exposure of a node. If nodes farther away are considered in the computation of the redundancy, the connected cover set will approach closer to the optimal value, i.e., it is expected to be smaller. This trade-off has been pointed out in.[21] Our solutions use only two states per node and use knowledge of nodes at a distance of up to two hops away. We conjecture that using only two states, it is impossible to construct a fully local (i.e., only the immediate neighborhood is known) self-stabilizing connected covers, hence our solutions use the minimal knowledge per node.

Our solutions do not require the sensors to have unique identifiers (ID's). However, each sensor i maintains a set of distinct labels, denoted as N_i, such that each label identifies a (unique) neighbor of i. Note that these labels are unique only *locally*.

The query region forms a convex region, and its boundary (hence, its center which is used by our first implementation) is known to all sensors. The energy level of the sensors may change over time due to various reasons. The proposed solutions cope with that.

We distinguish three types of sensors in or around the query region R_Q. In our algorithms, the rules for these three types of nodes are different. A sensor is termed as a *boundary sensor* if its sensing region intersects with the boundary of the query region R_Q. A sensor is called an *interior sensor* if its sensing region is completely inside the query region R_Q and it is not a boundary sensor. All sensors which are neither boundary nor interior are called *exterior sensors*. In other words, if a sensor's sensing region is completely outside the boundary of the query region, then this sensor is called an *exterior sensor*.

Data Structures. Three variables (R_Q, N_i, and Pos_i) are used as **Constants** by the proposed solutions. That is, the algorithms do not write into these variables. The input query includes the geographical information about the region R_Q to be covered. N_i represents the neighboring sensors of sensor i. Our solutions assume that there is an underlying self-stabilizing topology maintenance protocol which computes N_i. The sensors use either some device or/and protocol to know their geometric location.

Module 3.1 Data structures used by Cover Algorithms (Algorithms $\mathcal{DMSC}$ and $\mathcal{IMSC}$).

```
Constants:
    $R_Q$:: Query region; represented as a set in the algorithm;
    $N_i$ :: Set of sensors within the communication range of $i$;
    $Pos_i$ :: Geometric location or coordinates of Sensor $i$;
Shared Variables:
    $S_i$ :: Sensing region of Sensor $i$;
    $Color_i \in \{black, red\}$ :: Color of Sensor $i$;
Macros:
    $Dst(i) \equiv$ Returns the distance of $i$ from the center of $R_Q$ uncovered by i sensing region
        (used only in Algorithm $\mathcal{DMSC}$); uses $R_Q$ and $Pos_i$ to compute the distance;
    $BidirN(i) \equiv \{j | j \in N_i \wedge i \in N_j\}$;
    $RealBlackN_i :: \{j | j \in BidirN_i \wedge (Color_j = Black)\}$;
    $RealBlackN_{i,t} :: \{j | j \in BidirN_i \bigcup BidirN_t \wedge (Color_j = Black)\}$;
    $BlackConNbrs(i) \equiv \forall j, k \in RealBlackN_i : \exists l_1, \ldots, l_n \in RealBlackN_i : l_1 \in RealBlackN_j \wedge$
        $(\forall x < n : l_{x+1} \in RealBlackN_{l_x}) \wedge l_n \in RealBlackN_k$;
    $BlackConNbrs(i,t) \equiv \forall j, k \in RealBlackN_{i,t} : \exists l_1, \ldots, l_n \in RealBlackN_{i,t} : l_1 \in RealBlackN_j \wedge$
        $(\forall x < n : l_{x+1} \in RealBlackN_{l_x}) \wedge l_n \in RealBlackN_k$;
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The algorithms also use two **Variables**: S_i and $Color_i$. S_i represents the sensing region of sensor i. The $Color$ variable is used to represent the different status of a sensor. Sensors can be either in *red* or *black* initially. Eventually, if a sensor turns *black* and stays in that color, then it is considered to be a member of $\mathcal{MCSC}_Q$. Other sensors will remain in *red* color.

Macros. The macros do not represent variables, but return values when referred to in the code. Our first solution assumes that the sensors know the location of the center. So, they can use their location information (Pos) to compute their distances to the center. Note that this information is used uniquely by the first solution. The macro $Dst(i)$ computes the distance of Sensor i from the center of R_Q uncovered by the sensing region of i. We consider directed communication graph of sensors. So, a sensor i may not have a two-way communication with all its neighbors. Sensor i may need this knowledge (i.e., which of its neighbors have a two-way communication with it) to check redundancy. The macro $BidirN(i)$ returns a set of neighbors of i that effectively have a bidirectional communication with i. The predicate $BlackConNbrs(i)$ is used to implement the redundancy checking. This predicate is the local version of the "Rule k" of.[21] It returns true if the subgraph of black neighbors of a node is connected. We also introduce a novel version of the "Rule k", $BlackConNbrs(i,t)$, that checks if the subgraph defined by the black neighbors of two neighboring nodes, i and t, is connected.

IV. Distance-Dependent Query Cover

The first solution to the connected sensor cover problem is given as Algorithm 4.1 (referred in this paper as Algorithm $\mathcal{DMSC}$). The proposed algorithm starts from the boundary sensors of R_Q, and proceeds towards the center of the region. Starting from any configuration, the algorithm selects a few sensors among many (due to our assumption of very dense network) to include in the (minimum) cover set $\mathcal{MCSC}_Q$.

If the system starts from a good initial configuration, it first selects some boundary sensors, then some interior sensors, and keeps repeating the process of selection moving towards the center of the query region until it covers the whole region R_Q. Our solution is localized, meaning the decision to be selected in $\mathcal{MCSC}_Q$ is taken locally by all nodes. So, unlike the solution in,[9] nodes do not collect any global information to compute $\mathcal{MCSC}_Q$. Sensors consult only their immediate neighbors to decide if they should be included in the final set cover.

We also cannot adopt an aggressive procedure to improve our solution in terms of the size of the cover because that may waste a lot of sensor energy, which would conflict with the main goal of the research.

Algorithm 4.1 Self-Stabilizing Distance-Dependent Connected Sensor Cover Algorithm (Algorithm $\mathcal{DMSC}$) for Sensor i.

```
Predicates:
    $Boundary(i) \equiv S_i$ is a boundary sensor;
    $Interior(i) \equiv S_i$ is an interior sensor;
    $Exterior(i) \equiv S_i \cap R_Q = \emptyset$; is an exterior sensor;
    $Redundant(i) \equiv BlackConNbrs(i) \wedge (S_i \bigcap R_Q) \subseteq \bigcup_{j \in RealBlackN_i} S_j$;
    $SameIntrsctn(i, j) \equiv Interior(i) \wedge Interior(j) \wedge (\exists x, y \in \{N_i \cap N_j\}$:
        $(Color_x = Color_y = black) : (i \in N_x \wedge i \in N_y) \wedge Pos_i \in (S_x \cap S_y) \wedge Pos_j \in (S_x \cap S_y))$;
    $BestCandidate(i) \equiv \forall j \in N_i : SameIntrsctn(i, j) : (Dst(i) \leq Dst(j))$;
Actions:
    $\mathcal{A}_1$ :: $Boundary(i) \wedge \neg Redundant(i) \wedge Color_i \neq black \longrightarrow Color_i = black$
    $\mathcal{A}_2$ :: $(Redundant(i) \vee Exterior(i)) \wedge Color_i \neq red \longrightarrow Color_i = red$
    $\mathcal{A}_3$ :: $BestCandidate(i) \wedge \neg Redundant(i) \wedge Color_i \neq black \longrightarrow Color_i = black$
```

A. Normal Behavior

In the following, we assume that the system starts from a good initial configuration, meaning, all sensors are *red* initially. If the system starts from a good initial configuration, it first selects some boundary sensors, then some interior sensors, and keeps repeating the process of selection moving towards the center of the query region until it covers the whole region R_Q. In the following, we will first describe how some boundary *red* sensors are selected to turn *black* to cover the boundary of R_Q. Then we discuss the general case of covering any uncovered region inside the query region.

Boundary of R_Q. The initial task is to select enough sensors to cover the boundary with a communication network of sensors. The selected sensors will be colored *black*, and the rest will remain *red*. This is implemented in Action $\mathcal{A}_1$ using the predicates *Boundary* and *Redundant*. A boundary node will turn *black* and remain *black* only if it is not a redundant node. The redundant nodes will be eventually marked *red*. The redundancy is checked by using the predicate *Redundant*, and is described in detail in the next paragraph. Action $\mathcal{A}_1$ changes a sensor from *red* to *black*. Instead of wasting energy and time, we added the redundancy check in Action $\mathcal{A}_1$ itself. As we are using an asynchronous model, some nodes may be slow in executing Action $\mathcal{A}_1$, while their neighbors have already changed to *black* by executing the same action. So, the slow nodes soon after turning to *black* may find out that they are redundant. Then, they will have to turn to *red* by executing another action ($\mathcal{A}_2$). Note that, a *red* sensor turns *black* only after checking for possible redundancy in the neighborhood.

The above redundancy tests are implemented by a sensor i before it decides to withdraw itself from further consideration into the set cover $\mathcal{MCSC}_Q$. However, those tests only verify the coverage of i by other sensor (s). They do not implement the test of connectivity of the neighborhood of i. Recall that the set $\mathcal{MCSC}_Q$ must be a connected set cover. So, before removing itself, Sensor i wants to secure the connectivity in its neighborhood. This is implemented in the predicate $BlackConNbrs(i)$ which is a part of redundancy checking. Ideally, Sensor i needs to check if every pair of its back neighbors j and k, will remain connected if i is marked redundant and removed. However, if the path between j and k contains any node $l \notin RealBlackN_i$, then i cannot verify this path because our solution is strictly localized. So, our implementation of $BlackConNbrs(i)$ verifies if j and k are connected using some intermediate nodes $l_1, \ldots, l_n$ where all the intermediate nodes are black neighbors of i.

Inside R_Q. Current *black* nodes (their creation is discussed in the next paragraph) are used to create more *black* nodes to gradually cover the uncovered region. Future *black* nodes are selected from the intersections of pairs of existing *black* nodes. The algorithm considers every intersection of two *black* nodes. It chooses one or more *red* nodes from the intersection as the new members of the cover set $\mathcal{MCSC}_Q$ (using the predicate *BestCandidate*).

Note that the current *black* nodes may have been created using the boundary sensors selected earlier. Or, they were created by using the predicate *BestCandidate* among some sensors inside an intersection of two other *black* nodes.

Newly selected nodes for the cover set create a new (virtual) boundary of the uncovered query region. Thus, the algorithm reduces the uncovered portion of the query region R_Q by effectively pulling the (virtual) boundary towards the center of the query region. A sensor i is a possible candidate if it is located inside the intersection of the sensing regions of two other black sensors. As we are dealing with dense sensor network, lots of redundant sensors are expected to be in every intersection of two black sensors. The algorithm uses some checking ($Redundant$) (explained above), and marks the best candidates as $black$. The best candidates are those which are closest to the center of R_Q among all the candidates in a particular group. The reason for using this distance to remove redundancy is that the algorithm covers the query region starting from the boundary towards the center. The candidates compare their distances to the center using the predicate $BestCandidate$. Eventually, every intersection between two $black$ nodes will be in one of the following two situations:

1. Node i is the best candidate in its neighborhood inside the intersection because it is the nearest node to the center of R_Q (see $BestCandidate$). If i is not a redundant node, it will execute Action $\mathcal{A}_3$ and turn $black$.
2. Sensor i is one of the sensors (let us refer them by a set B) (all in the same intersection) at the same shortest distance from the center of R_Q. In this case, all nodes in B will satisfy $BestCandidate$ and turn $black$ (provided they are not redundant) by executing Action $\mathcal{A}_3$. Then B is the set of best candidates to be included into $\mathcal{MCSC}_Q$.

B. Faults and Recovery

In this section, we focus on the fault handling features of the proposed algorithm (Algorithm 4.1). There are two variables used in the solution: S_i and $Color_i$ for a Sensor i. So, we need to show that our solution can cope with all possible corruptions associated with these two variables.

1. Wrong initialization of the color variable. As discussed in Section A, all sensors, if properly initialized start as red.
   - Corruption of a boundary sensor. Assume that a boundary sensor i starts in $black$ color. If i is not a redundant node, then i remains $black$ (see Action $\mathcal{A}_1$). That is, no correction is necessary. If i is redundant, then it will satisfy the predicate $Redundant$, hence the guard of Action $\mathcal{A}_2$. Node i then executes $\mathcal{A}_2$ and changes its color to red.
   - Corruption of an interior sensor. Assume that an interior node is initialized as a $black$ colored sensor. If i is not a redundant node, then i remains $black$ (see Action $\mathcal{A}_2$). So, no correction is necessary. If i is redundant, then it will satisfy the predicate $Redundant$ and execute Action $\mathcal{A}_2$ which will change its color to red.
   - Corruption of an exterior sensor. All exterior sensors must be eventually colored red. If any exterior sensor starts as $black$ initially, then it will execute Action $\mathcal{A}_2$ to change its color to red.
   - Best candidate sensor's color is corrupted from $black$ to red. Action $\mathcal{A}_3$ corrects the color back to $black$.
   - A redundant sensor's color changes from red to $black$. The node, regardless of whether boundary or interior, will satisfy $Redundant$ and hence, the guard of Action $\mathcal{A}_2$. So, it will change its color to red.
2. Weakening or Failure of sensors, both in terms of communication and sensing ability. The weakening or failure of sensors will affect the sensing and communication range of the sensors. In other words, the constant set N and the variable S will change. Change of S may change the values of $Redundant$ and $BestCandidate$. All these changes will be reflected in the change of values of the guards. So, eventually, the color of the affected node will change due to the execution of the actions. All change of colors have already been discussed in earlier cases above.

C. Proof of Convergence

In this section, we will prove the convergence of Algorithm $\mathcal{DMSC}$ (presented in Section IV). That is, our task is to prove that the proposed solution to the connected sensor cover problem satisfies the given specification. The outline of this section is as follows: We will first define a legitimacy predicate of Algorithm $\mathcal{DMSC}$ with respect to the specification of the proposed problem. Next it will be shown that the algorithm is guaranteed to arrive at a legitimate state regardless of the initial configuration or type of faults occurring in the system.

Definition 4.1 (Legitimacy Predicate $\mathcal{L}_{\mathcal{DMSC}}$). *The system is considered to be in a legitimate state (i.e., satisfies the legitimacy predicate $\mathcal{L}_{\mathcal{DMSC}}$) if the following conditions are true with respect to a query region:*
(i) All non-redundant boundary sensors are $black$. (ii) All non-redundant best candidate sensors are $black$. (iii) All other active sensors—exterior, boundary, and interior—are red.

Our obligation in this section is to prove that starting from any arbitrary configuration of the system of sensors, Algorithm $\mathcal{DMSC}$ guarantees that in finite steps, the system will reach a configuration that satisfies the legitimacy predicate $\mathcal{L}_{\mathcal{DMSC}}$. The proof outline is as follows: We first show that starting from an arbitrary configuration, the boundary of R_Q will be covered. Then we establish the progress towards covering the whole region by proving that every $black$ node creates two more $black$ nodes to cover some other uncovered area of R_Q. This process is repeated until R_Q is completely covered.

Lemma 4.1. *Starting from any arbitrary configuration, the boundary of the input query R_Q will be covered.*

Proof. By contradiction. Assume that there is an area A which intersects the boundary of R_Q is not covered. A must contain at least an active sensor i. Then if Action $\mathcal{A}_1$ is enabled at i, i will turn $black$. Considering other sensors in A, A will eventually be covered. That is a contradiction. Let us assume that i is not enabled to execute Action $\mathcal{A}_1$. Then per guard of $\mathcal{A}_1$, there are two possibilities:

1. Sensor i is $black$. So, considering other sensors (like i) in A, A is covered. That is a contradiction.
2. The predicate $Redundant(i)$ is true. By the definition of $Redundant(i)$, it follows that i is covered by $black$ sensors. Again, considering other active sensors in A, we obtain that A is covered, hence the contradiction. □

Lemma 4.2. *In any configuration, if all boundary and interior red nodes are redundant, then the region R_Q is completely covered.*

Proof. Assume the contradictory, i.e., although all boundary and interior red nodes satisfy $Redundant$ predicate, R_Q is not completely covered yet. Consider an area A intersecting R_Q which is not covered. A must contain at least an active sensor i. The color of i cannot be $black$ since A is assumed to be uncovered. So, i is red. Since A is not covered, i will not satisfy $Redundant(i)$. That contradicts the lemma hypothesis. □

Lemma 4.3. *Every $black$ node covering a region of R_Q will eventually add two neighboring $black$ nodes unless the new nodes are found to be redundant.*

Proof.
1. Consider a $black$ boundary node r. That is, r covers a region on the boundary of R_Q. The existence of at least one such node is implied by Lemma 4.1. Assume that r has two $black$ neighbors, r_1 and r_2. This is a valid assumption because only one $black$ sensor covering a region is very unlikely. Let I_1 be the area of intersection between the sensing regions of r_1 and r. Then in I_1, there must exist a node p that satisfies $Best\_candidate(p)$. So, unless p is redundant (i.e., satisfies the predicate $Redundant(p)$), it will execute Action $\mathcal{A}_3$ to turn $black$. Similarly, let I_2 be the intersection between the sensing regions of r and r_2. So, there must be a sensor q in I_2 which will satisfy $Best\_candidate(q)$, perform $\mathcal{A}_3$, and change its color to $black$ if it is not a redundant node. Note that both p and q are interior nodes.
2. Now, consider a $black$ interior node r covering a region in R_Q. By Lemma 4.2 and the above case, a node like r exists unless R_Q is completely covered. Following the same reasoning as in the above case, we can show that r will add two more $black$ nodes unless the new nodes are marked to be redundant. □

Lemma 4.4. *Starting from an arbitrary configuration, the input query region R_Q will eventually be completely covered by $black$ nodes.*

Proof. Assume that $Covered\_Region_i$ and $Uncovered\_Region_i$ represent the current *covered* and *yet to be covered* region of R_Q, respectively. By Lemmas 4.2 and 4.3, there must exist at least one $black$ node r in $Covered\_Region_i$, which will generate two more $black$ sensors. These new $black$ sensors will cover some portion of $Uncovered\_Region_i$, effectively reducing the area of $Uncovered\_Region_i$. Therefore, repeated application of Lemma 4.3 progressively reduces the area of $Uncovered\_Region_i$. Since $Uncovered\_Region_i$ is finite, eventually the system reaches a configuration which satisfies one of the following two conditions:

1. $Uncovered\_Region_i$ becomes an empty set. That is, R_Q is completely covered.
2. $Uncovered\_Region_i$ is nonempty. By the lemma hypothesis, there are some *black* nodes in $Covered\_Region_i$. By Lemma 4.3, a *black* node will create two more *black* nodes. If the newly created nodes are redundant, then by Lemma 4.2, R_Q is already covered. But, that is a contradiction. □

Theorem 4.1 (Convergence). *Starting from an arbitrary configuration, Algorithm $\mathcal{DMSC}$ reaches a configuration that satisfies the legitimacy predicate $\mathcal{L}_{\mathcal{DMSC}}$.*

Proof. By Lemma 4.4, R_Q will be eventually covered. Starting from this configuration, we now prove that the system will reach a configuration satisfying $\mathcal{L}_{\mathcal{DMSC}}$. In the following, we will consider the three conditions to be satisfied to satisfy $\mathcal{L}_{\mathcal{DMSC}}$.

(i) *All non-redundant boundary sensors are black.*
The proof follows from Action $\mathcal{A}_1$.

(ii) *All non-redundant best candidate sensors are black.*
The proof follows from Action $\mathcal{A}_3$

(iii) *All other active sensors—exterior, boundary, and interior—are red.*
Exterior nodes will turn *red* by applying Action $\mathcal{A}_2$. Other nodes if redundant will turn *red* by executing Action $\mathcal{A}_2$. The nodes which are not best candidates (if not already redundant) will eventually become redundant when they will be covered by *black* nodes, and will turn *red* (if not already *red*). □

V. Distance-Independent Query Cover

In the previous section, we proposed a distance-based solution to covering a query region. In this section, we present a solution which does not need to compute the center of the query region. The proposed algorithm given as Algorithm 5.1 (referred in this paper as Algorithm $\mathcal{IMSC}$) has a different approach than Algorithm $\mathcal{DMSC}$. It chooses non-redundant nodes inside the query region. Note that in a good initial configuration (when all nodes are *red*), every node inside the query region is a potential candidate to be selected in the final cover $\mathcal{MCSC}_Q$. Our algorithm removes the redundant nodes from the $\mathcal{MCSC}_Q$. The two phases of the algorithm are executed based only on the local information available to the nodes.

Algorithm 5.1 Self-Stabilizing Distance-Independent Connected Sensor Cover Algorithm (Algorithm $\mathcal{IMSC}$) for Sensor i.

```
Predicates:
    $Useless(i) \equiv (BidirN_i = \emptyset);$
    $Exterior(i) \equiv (S_i \cap R_Q) = \emptyset);$
    $Redundant(i) \equiv BlackConNbrs(i) \wedge (S_i \bigcap R_Q) \subseteq \bigcup_{j \in RealBlackN_i} S_j;$
    $Potential\_Candidate(i) \equiv \neg(Useless(i) \vee Exterior(i)) \wedge \neg Redundant(i);$
    $Candidate(i) \equiv Potential\_Candidate(i) \wedge (\forall j \in BidirN_i : Potential\_Candidate(j) :: i < j)$
    $Potential\_Link(i, j, k) \equiv Potential\_Candidate(i) \wedge \neg Candidate(i) \wedge$
        $(\exists t \in Bidir(i), \exists j, k \in RealBlackN_i \bigcup RealBlackN_t, \neg(BlackConNbrs(i) \vee BlackConNbrs(i, t))$
    $Link(i) \equiv \exists j, k, Potential\_Link(i, j, k) \wedge (\forall s : Potential\_Link(s, j, k) :: i < s)$
Actions:
    $\mathcal{A}_1 : (Candidate(i) \vee Link(i)) \wedge Color_i \neq Black \rightarrow Color_i = Black;$
    $\mathcal{A}_2 : \neg(Candidate(i) \vee Link(i)) \wedge Color_i \neq Red \rightarrow Color_i = Red;$
```

A. Normal Behavior

Starting from a good configuration, all candidates or links eventually become *black* by executing $\mathcal{A}_1$. A process is a *candidate* if it is inside the query region or on the boundary, but not a redundant node. A node is *critical* if removing it disconnects at least a pair of its neighbors locally. Due to the asynchronous execution of the algorithm, all nodes may become candidates at the same time. In order to break the symmetry, only one node can be a candidate in any neighborhood. However, choosing one candidate per neighborhood may disconnect the cover. This is avoided by using *link nodes*. A node is a *potential link* node between two black nodes if it is a potential candidate but not a candidate, and the two black nodes are not neighbors of each other. That is, a potential link node works as a bridge between two black nodes that would otherwise be disconnected. A *link* node between two black nodes is the node with the minimum ID (as per the local label) among the potential link nodes of the black nodes.

Since, rule $\mathcal{A}_1$ is applied asynchronously it is possible that redundant links and candidates are created. Rule $\mathcal{A}_2$ removes all redundancy in the connected cover.

B. Faults and Recovery

As in Algorithm 4.1, this algorithm also has two variables used in the solution: S_i and $Color_i$ for a Sensor i. So, we need to show that our solution can cope with all possible corruptions associated with these two variables.

1. Wrong initialization of the color variable. In Algorithm $\mathcal{IMSC}$, all sensors, if properly initialized start as *red*.
   - Corruption of the interior & boundary Sensors. Assume that an interior or a boundary node is initialized as a $black$ sensor. Let i be this node. If i is not a redundant node or it is critical, then i remains *black* (see Action $\mathcal{A}_2$). So, no correction is necessary. If i is redundant, then it executes Action $\mathcal{A}_2$ which will change its color to red.
   - Corruption of the exterior sensors & useless sensors. All exterior sensors must be eventually colored red. If any exterior sensor starts as $black$, then it will execute Action $\mathcal{A}_2$ to change its color to red. If a $black$ sensor cannot be sensed by none of its neighbors, then it will turn red by executing $\mathcal{A}_2$.
   - Candidate or link sensor's color is corrupted from $black$ to red. Action $\mathcal{A}_1$ corrects the color back to $black$.
   - Non-candidate or non-links sensor's color changes from red to $black$. The node satisfies the guard of Action $\mathcal{A}_2$. So, it will change its color to red.
2. Weakening or Failure of sensors, both in terms of communication and sensing ability. The algorithm behaves as Algorithm $\mathcal{DMSC}$ in this situation.

C. Proof of Convergence

The legitimate configuration definition for Algorithm $\mathcal{IMSC}$ is based on the notion of candidate.

Definition 5.1 (Legitimacy Predicate $\mathcal{L}_{\mathcal{IMSC}}$). *The system is considered to be in a legitimate state (i.e., satisfies the legitimacy predicate $\mathcal{L}_{\mathcal{IMSC}}$) if the following conditions are true with respect to a query region:*

(i) All candidate and link sensors are black.
(ii) All other active sensors—exterior, boundary, and interior—are red.

In the following, we prove that starting in an arbitrary configuration, the algorithm converges to a legitimate configuration satisfying Definition 5.1.

Lemma 5.1. *Starting from an arbitrary configuration, each node changes its color a finite number of times.*

Proof. Any exterior node or useless node executes at most one step. If the node is black then it turns red, otherwise it does not change its color.

A red interior node may execute first $\mathcal{A}_1$ and turns black then it may execute $\mathcal{A}_2$ and turns again red. In the following we show that the previous scenario can be repeated only a finite number of times. Such a scenario is possible if the node is part of a cycle that includes oscillating nodes. Assume the contrary. There is no cycle of oscillating nodes that contains the node. Hence we can construct a linear chain formed only with oscillating nodes. This chain will have at least one node outside the query region or at least one inside node that eventually will not oscillate. After a finite number of steps the chain eventually stabilizes. That is, if the oscillating node is part of an acyclic structure then after a finite number of steps the chain stabilizes.

Let $p_1 \dots p_l \dots p_1$ be the oscillating cycle. The cycle oscillates only if: all nodes execute rule $\mathcal{A}_1$, then all of them execute $\mathcal{A}_2$ and so on. This scenario is not possible because of the Candidate predicate. In a neighborhood only one node has its predicate verified, hence the symmetry is broken. □

Lemma 5.2. *Starting from an arbitrary configuration, eventually, the input query region will be completely covered by $black$ nodes.*

Proof. Let c be the initial illegitimate configuration. Each node in the query region executes a finite number of times Algorithm $\mathcal{IMSC}$ (Lemma 5.1). Hence there exists c' a configuration such that no node in c' is able to execute its algorithm. Assume that such configuration does not exist. Hence a node executes forever its algorithm. Let c'' be the configuration where all the nodes but one, p, are disabled.

Let $Cover$ be the query region covered by black nodes in c''. If p is candidate or link node colored red in c'' . then p executes rule $\mathcal{A}_1$ and colors itself black. That is, after the p execution the query region covering is completed.

If p is a redundant node and not critical then p executes $\mathcal{A}_2$ and colors itself in red. Again, after the p's execution the query region covering is completed. □

Theorem 5.1 (Convergence). *Starting from an arbitrary configuration, Algorithm $\mathcal{IMSC}$ reaches a configuration that satisfies the legitimacy predicate $\mathcal{L}_{\mathcal{IMSC}}$.*

Proof. By Lemma 5.2, R_Q is eventually covered. In the following, we will consider the two conditions to be satisfied to satisfy $\mathcal{L}_{\mathcal{IMSC}}$.

(i) *All non-redundant boundary and interior sensors are $black$.*
The proof follows from Action $\mathcal{A}_1$.

(ii) *All other active sensors—exterior, boundary, and interior—are red.*
Exterior nodes will turn red by applying Action $\mathcal{A}_2$. Other nodes if redundant will turn red by executing Action $\mathcal{A}_2$. □

VI. Self-∗ Proofs

We want to conclude the proof of Algorithms $\mathcal{DMSC}$ and $\mathcal{IMSC}$ in this section by showing how our solutions satisfy some of the self-∗ properties. Algorithms $\mathcal{DMSC}$ and $\mathcal{IMSC}$ are distributed, self-configuring, self-healing, and scalable. Thus, the proposed self-∗ solution makes the goal of ubiquitous/pervasive computing a reality since two of the main requirements for this type of large ubiquitous sensor networks are low-power and self-configuring.

The next two lemmas prove that the two proposed algorithms satisfy the coverage and connectivity properties of the connected cover problem. These two results will be used later to prove some self-∗ properties.

Lemma 6.1 (Coverage). *In any legitimate configuration, the connected set cover $\mathcal{MCSC}_Q$ computed by Algorithm $\mathcal{DMSC}$ or Algorithm $\mathcal{IMSC}$ completely covers the query region R_Q.*

Proof. By contradiction. Assume that there is an area A which intersects R_Q is uncovered by $\mathcal{MCSC}_Q$. Since the network is assumed to be densely deployed, A must contain at least an active sensor i which is obviously *red*. Then according to the predicates $\mathcal{L}_{\mathcal{DMSC}}$ or $\mathcal{L}_{\mathcal{IMSC}}$, i must be a redundant sensor. By the definition of predicate $Redundant(i)$, i must be covered by some $black$ nodes. Since i was chosen to be any node in the uncovered area A, we can claim that all active red sensors in A are covered by some $black$ sensors. Therefore, A is covered, and we arrive at the contradiction. □

Lemma 6.2 (Connectivity). *In any legitimate configuration, the connected set cover $\mathcal{MCSC}_Q$ computed by Algorithm $\mathcal{DMSC}$ or Algorithm $\mathcal{IMSC}$ forms a connected graph.*

Proof. By contradiction. Assume that there exist two node-disjoint connected components in the set $\mathcal{MCSC}_Q$. All active sensors initially form a connected graph. So, the only way for the set being disconnected is by marking some active sensor (say i) as redundant such that not considering i as part of the final set $\mathcal{MCSC}_Q$ disconnected i's neighborhood.

However, per predicate $BlackConNbrs(i)$, i is considered to be a redundant node only after ensuring the complete bidirectional connectivity of its neighborhood. That is, if i was marked redundant and colored red by Action $\mathcal{A}_2$, all neighbors of i remained connected. In other words, if the set $\mathcal{MCSC}_Q$ was connected before $\mathcal{A}_2$ was executed, it would remain connected after the execution of the action as well. We reach the contradiction. □

Self-configuring and Self-healing. Algorithms $\mathcal{DMSC}$ and $\mathcal{IMSC}$ are self-configuring in the sense that starting from any initial configuration, they configure themselves to form a network topology where all sensors (members of the connected sensor cover) are able to communicate with each other either directly or indirectly. We also proved that the given query region will eventually be covered starting from any arbitrary state. Hence these algorithms are self-configuring.

The proposed algorithms are self-healing under various perturbations, such as node joins, failures (due to crash and energy loss), state corruptions, and weakening of power. If a node fails, Algorithms $\mathcal{DMSC}$ and $\mathcal{IMSC}$ heal themselves in the following manner: If it is not a redundant node, then a part of the query region R_Q may become uncovered. In that situation, a subset of its (active) neighbors will take over by executing Action $\mathcal{A}_1$ or $\mathcal{A}_3$ in Algorithm $\mathcal{DMSC}$, and $\mathcal{A}_1$ in Algorithm $\mathcal{IMSC}$. A similar process may be initiated, if necessary, when a node's power weakens to the point that it affects the node's communication ability. On the other hand, if a node joins the network (after recovering, being repaired, or being re-energized in power), it would be considered as a redundant node since the query region is already covered by the existing nodes. So, the node joining event will not change the connected set cover. Arbitrary corruptions of state variables of the nodes are also dealt with in the solution—change of $Color$ variable due to faults is fixed in a very simple manner.

Self-stabilization.

Theorem 6.1 ($\mathcal{L}_{\mathcal{DMSC}}$ and $\mathcal{L}_{\mathcal{IMSC}}$ satisfy specification). *Any system configuration satisfying the legitimacy predicates $\mathcal{L}_{\mathcal{DMSC}}$ or $\mathcal{L}_{\mathcal{IMSC}}$ (per Definitions 4.1 or 5.1, respectively) satisfies the specification of the minimal*[†] *connected sensor cover problem (per Definition 3.2).*

Proof. The coverage and connectivity properties are proved in Lemmas 6.1 and 6.2, respectively. The definitions of $\mathcal{L}_{\mathcal{DMSC}}$ and $\mathcal{L}_{\mathcal{IMSC}}$ imply that in a legitimate configuration no node can be removed from the coverage sets. Therefore, the connected cover set $\mathcal{MCSC}_Q$ computed at this point are minimal. □

Lemma 6.3 (Closure). *The legitimacy predicates $\mathcal{L}_{\mathcal{DMSC}}$ and $\mathcal{L}_{\mathcal{IMSC}}$ are closed.*

Proof. In any configuration satisfying $\mathcal{L}_{\mathcal{DMSC}}$ or $\mathcal{L}_{\mathcal{IMSC}}$, all actions of Algorithm $\mathcal{DMSC}$ or Algorithm $\mathcal{IMSC}$ respectively are disabled. Therefore, the algorithms are silent, and satisfy the closure property. □

Theorem 6.2. *Algorithms $\mathcal{DMSC}$ and $\mathcal{IMSC}$ are self-stabilizing.*

Proof. The proof follows from Theorem 6.1, Lemma 6.3, and Theorems 4.1 and 5.1. □

VII. Complexity and Simulation

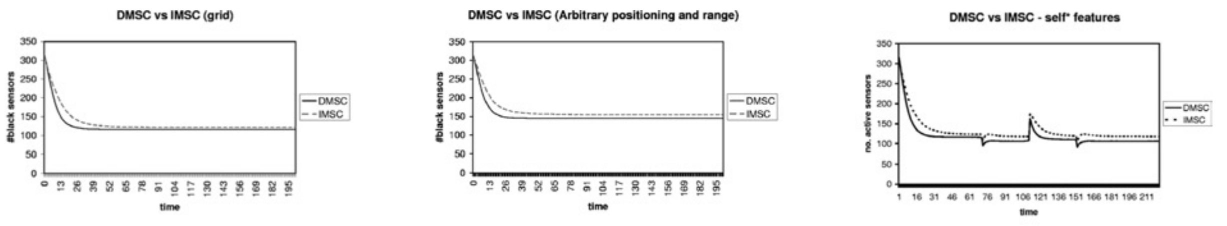
Algorithms $\mathcal{DMSC}$ and $\mathcal{IMSC}$ are space *optimal* solutions. Recall the problem specification. Upon termination of the sensor cover algorithm, a node will know if it should be active or passive. So, it must use at least two states to distinguish its two possible roles. Our solutions use exactly two states in the $Color$ variable to implement this. Moreover, both solutions compute a minimal cover for the query region.

In our simulations, for the first set of experiments, we assumed that nodes are fixed and uniformly deployed on a grid of size 25 × 25 (625 nodes). Similar to[9,12,15] we consider the sensing region associated with a sensor modeled as a circular region around itself. We considered a homogeneous network (i.e. all sensors have the same sensing region — circular of radius 9). Then in the second set of experiments, we considered 625 sensors randomly deployed in a square region of size 100, with different sensing regions uniformly distributed in the range between 5 and 15. In all the simulations, we considered a circular query region of radius 40. The number of experiments performed for each simulated point was 100. We also performed simulations of the Distributed Priority algorithm of.[19] The performance metrics we measured are the stabilization time and the number of nodes per cover. The average results

[†] A connected cover is minimal if it does not include another connected cover.

Table 1 Average performance metrics.

Parameter	$\mathcal{DMSC}$	$\mathcal{IMSC}$	19
Self-stabilization capabilities	Yes	Yes	No
Self-stabilization Time - grid deployment range 9	41	64	Not self-stabilizing
Self-stabilization Time - random deployment range 9	32	91	Not self-stabilizing
Self-stabilization Time - random deployment range 5–15	41	160	Not self-stabilizing
Cover Size - grid deployment range 9	116	123	145
Cover Size - random deployment range 9	146	157	204
Cover Size - random deployment range 5–15	145	154	250



(a) Grid - homogeneous sensing range 9. (b) Arbitrary placement, heterogeneous sensing range 5-15 (c) Self-* in the presence of transient and permanent faults

Fig. 1 Experimental results.

are summarized in Table 1. Since algorithm[19] is not self-stabilizing, the parameter "self-stabilization time" cannot be computed for this particular algorithm.

Our experiments for both the topologies (grid and arbitrary) show that Algorithms $\mathcal{DMSC}$ and $\mathcal{IMSC}$ have almost identical behaviors in terms of the coverage size, Algorithm $\mathcal{DMSC}$ computing a slightly better cover.

We can also observe that in terms of the cover size, the algorithm[19] is inferior to ours. The main reason for this is the mechanism of removing the redundant nodes. In our solutions, the redundant nodes are removed by checking the existence of a connectivity path in the set of "black" neighbors, while in their solution, the redundant nodes are removed by checking the existence of a connectivity path only in the set of black neighbors with lower priority.

Algorithm $\mathcal{DMSC}$ stabilizes faster than Algorithm $\mathcal{IMSC}$ (see Figs. 1 a, b, c). The slower stabilization of Algorithm $\mathcal{IMSC}$ is due to the fact that a sensor inside the query region can change its color more than once. In Algorithm $\mathcal{DMSC}$, a sensor inside the query region waits until the covering wave arrives in its neighborhood.

Interestingly, the type of sensing regions (homogeneous or different) and the placement of sensors (grid or arbitrary) does not make a strong impact on the coverage size and stabilization time for Algorithm $\mathcal{DMSC}$. Algorithm $\mathcal{IMSC}$ seems to be more sensitive to the topology and sensors characteristics.

We performed additional simulations for a densely populated environment, 500 sensors uniformly deployed on a grid of size 40×40. In the case of the random placement and a sensing region of size 9, the average results confirm the previous observations (i.e., our algorithms are more efficient). That is, the size of the coverage set computed by algorithms $\mathcal{DMSC}$ and $\mathcal{IMSC}$ is on an average 35 sensors, while the size of the coverage computed by[19] is 38 sensors.

We also simulated the impact of various types of faults (crash and memory corruption) on the construction of the cover computed by IMSC and DMSC (Fig. 1c). The simulation results confirm the theoretical analysis. That is, our algorithms self-heal starting from any configuration including memory corrupted configurations.

The fault injection was performed following the here after scenario. At times 72 and 112, 30% of nodes experience crash and transient failures, respectively. Fig. 1c shows that the systems is able to self-heal. At time 152, 30% of black nodes (nodes that form the cover) experience memory corruptions (i.e., their color change from black to red). The system self-heals again without any external intervention.

VIII. Future work and Conclusion

The main motivation of our research was to design a totally distributed self-$*$ query response system in sensor networks. We have presented the first local, distributed, scalable, self-stabilizing designs of the connected sensor cover problem introduced in.[9] We presented two stabilizing solutions to the problem and showed how the solutions are self-organizing and self-healing as well. Algorithms are space optimal—only two colors are used. Once the system is stabilized, the faults can be corrected in their neighborhood. Hence the system is self-containing. This research showed that the concept of self-stabilization subsumes many other self-$*$ properties. We were also able to demonstrate the power of self-stabilization to achieve quite an elegant and efficient solution to a very practical problem in a large scale wireless sensor network.

The connected sensor cover problem is a global task, meaning nodes cannot locally compute the final response to the query. However, we still require the algorithm to be *local* in the sense that the nodes collect information from their immediate neighbors. Unlike the solution in,[9] no node in the proposed algorithm collects global information, and no node behaves as a special node in any stage of the execution of the algorithm. In our solution, every node can locally decide if it should be an active or passive node in the current computation of the response to the query. Moreover, unlike all the pre-existing solutions to this problem our solutions are able to locally cope with memory and program counter corruptions. In summary, we achieve a global objective by using local and self-stabilizing algorithms.

Sensing coverage characterizes the monitoring quality provided by a sensor network in a designated region. Different applications may require different degrees of sensing coverage. Our algorithms were designed with the goal of achieving optimality in terms of the number of sensors while maintaining the degree of coverage of only one. That is, even if a single active sensor fails, some area may become uncovered right away. Recall that the proposed solutions are self-stabilizing with respect to all types of transient faults. So, they will cover the uncovered area very quickly. Thus, no area will remain uncovered forever. In this regard, we can extend our solutions in a couple of ways. Firstly, we may write a parametric solution where the input query will include the degree of coverage expected. The predicate $Redundancy$ will be relaxed to allow the corresponding higher degree of coverage. Secondly, we can simply assume a particular degree (>1) of coverage in our algorithm. Similar to the implementation of a higher degree of coverage to achieve better robustness, we may also require a higher degree of connectivity for the same purpose (i.e., to increase the level of fault-tolerance). We can extend the neighborhood connectivity checking to k-node ($k > 1$) disjointness in the communication graph. Unfortunately, higher degree of coverage/connectivity would require more communication cost, i.e., consuming more power. We can conduct a study on the trade off between connected cover size optimality vs. robustness and energy efficiency. Another interesting way to extend our work is to implement different degrees of connectivity in the final communication graph produced by the connected sensor cover algorithm. In the current solution, the predicate $BlackConNbrs$ implements the connectivity. It guarantees that all the neighbors of a node i are strongly connected before marking i as redundant. As in the case of better coverage, we cannot ignore the power consumption. So, the above study would involve investigating the impact of better fault-tolerance on the energy spent.

Another approach to solving the connected sensor cover problem is based on Steiner trees. The algorithm would require two phases. In the first phase, a sensor cover is computed without considering the connectivity criterion. In the next phase, a Steiner tree is constructed on the sensor cover to connect the nodes in the cover set. This approach may have two negative impacts. First, the set cover produced finally at the end of the second phase may have more redundant nodes than the approach used in our solutions. Second, computing the Steiner tree may incur higher communication cost. However, we can study implementation of various types of heuristic methods used in constructing Steiner trees in our common setting—local, distributed, and self-stabilizing.

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